

■ Research Paper

Snakes all the Way Down: Varela's Calculus for Self-Reference and the Praxis of Paradise

André Reichel*

European Center for Sustainability Research, Zeppelin University, Friedrichshafen, Germany

This contribution seeks to commemorate Francisco Varela's formal conceptions of self-reference, providing an overview of his writings while shedding some light in the praxis of self-reference, from where a future research agenda can be derived. The architecture of Varela's thinking, determined by the interrelated notions of autopoiesis, autonomy, closure and self-reference, was examined. The emphasis was on the development and expansions of his calculus for self-reference from George Spencer Brown's *Laws of Form*. After dealing with some of the criticism launched at both works, an appraisal of the praxis of self-reference of Varela's thinking in action was given. The outlook rounds up this contribution, shedding some light on a possible future research agenda for the formalization of theory and praxis of self-reference. Copyright © 2011 John Wiley & Sons, Ltd.

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INTRODUCTION

If everybody would agree that their current reality is *a* reality, and that what we essentially share is our capacity for constructing a reality, then perhaps we could agree on a meta-agreement for computing a reality that would mean survival and dignity for everybody on the planet, rather than each group being sold on a particular way of doing things. Thus,

self-reference is, for me, the nerve of this logic of *paradise* ... (Varela, 1976: 31)

Francisco Varela died 10 years ago and left us with a rich body of work, radiating into many different fields and problem areas, probably beyond his dreams and hopes. This contribution seeks to commemorate Varela's formal conceptions of self-reference, providing an overview of his writings while shedding some light on the praxis of self-reference, from where a future research agenda can be derived. The issues discussed here will start from the original architecture of Varela's thinking across his 30

*Correspondence to: Dr André Reichel, European Center for Sustainability Research, Zeppelin University, Am Seemooser Horn 20, Friedrichshafen 88045, Germany.
E-mail: andre.reichel@zeppelin-university.de

years of scholarly life to the intertwined notions of autopoiesis, autonomy, closure and self-reference. We will then continue with his work on the formal aspects of self-reference as developed in *A Calculus for Self-reference* (Varela, 1975) and the extensions made in collaboration with Joseph Goguen and Louis Kauffman. The criticisms launched at this part of Varela's work, especially at its basis, the *Laws of Form* by George Spencer Brown (Spencer Brown, 1969), will be categorized and rebutted. In the final part, we will dive into some of the many applications of his thinking, namely, in psychology, his own theory of embodiment, and the application in social systems theory by Niklas Luhmann (1995a).

The frequent use of direct and unabbreviated quotations from Varela himself offers the reader the possibility to assort and evaluate his original ideas with the conclusions we are drawing from them. The main argument encountered again and again is that Varela provided a formal grounding for dealing with self-reference, regardless of the discipline one is affiliated with or the phenomenon at hand. This contribution demands an 'open heart' (Kauffman and Solzman, 1981: 256) and the willingness to dive into the logic of paradise.

THE AUTONOMY OF VARELA'S THINKING

What is striking about Francisco Varela and his work is the great coherence of its architecture. This architecture rests on four interrelated notions: autopoiesis, autonomy, closure and self-reference. All four notions reverberate throughout the 30 years of research Varela undertook, from the humble beginnings in Chile and the autopoiesis of living systems (Varela *et al.*, 1974) to his work on phenomenology and philosophy of biology (Weber and Varela, 2002). All of his work contributed, in one way or another, to the core question of biology, posed by Erwin Schrödinger: 'What is life?' (Schrödinger, 1944). Varela's attempt of an answer led him to a magical mystery tour deep into philosophical territory, in fact, 'into a more than 2500-year-old philosophical minefield about the nature of reality and cognition' (Brier, 1996: 231–232). This

quotation from Brier, although aiming at second-order cybernetics, is valid for the quest Varela was following, because life and cognition are so intricately intertwined that the entire notion of a 'cybernetics of cybernetics' only makes sense when connected to the living.

Starting with notably the most famous notion of the four, *autopoiesis*, its classic definition from the seminal paper by Varela *et al.* (1974: 188) indicates it as the production of a 'unity by a network of productions of components which (i) participate recursively in the same network of productions of components which produced these components, and (ii) realize the network of productions as a unity in the space in which the components exist.' The criteria for autopoiesis, that is, if we are in fact dealing with an autopoietic unit, have been simplified by Varela himself in his essay 'El fenómeno de la vida', from which Luisi (2003) quotes that they consist of 'verifying (1) whether the system has a semi-permeable boundary that (2) is produced from within the system and (3) that encompasses reactions that regenerate the components of the system.' In order to understand where autopoiesis comes from and what the motives of its 'parents' were, one has to recall the state of biology in the 1960s and 1970s. The dominant approach back then was gene-centrism, the idea that the living is determined by its genetic constitution, reducing Schrödinger's question to a mere counting of RNA, DNA and enzymes (Varela, 1996). The feeling of revolution in biology and maybe in the whole of science as the focus of interest started to shift 'from causal unidirectional to mutualistic systemic thinking, from a preoccupation with the properties of the observed to the study of the properties of the observer' (Howe and von Foerster, 1975: 1), coincided with the revolution in Chile's political system, signified by the election of Salvador Allende as president in 1970. All distinctions in science that are drawn to understand the objects of interest, in fact, all distinctions in all areas of life, are drawn with a motive, for there 'can be no distinction without motive, and there can be no motive unless contents are seen to differ in value' (Spencer Brown, 1969: 1). This idea that every distinction, and thus every concept in science, including its understandings and misunderstandings,

indicates its motive and cannot be understood properly without taking this into account will come up in this contribution from time to time.

Autonomy, turning to the second important notion, 'is the distinctive phenomenology resulting from an autopoietic organization: the realization of the autopoietic organization is the product of its operation' (Varela *et al.*, 1974: 188). Varela illustrates autonomy with the example of the immune system, which he views as autonomous due to its autonomous functioning within the organism. An autonomous system might not have a clear-cut boundary like a cell membrane, but it has a boundary drawn in operation, that is, in the way the system functions autonomously in relation to its environment (Varela, 1978: 80). Although he draws a distinction between autopoiesis and autonomy, restricting autopoiesis to biology and living systems, thus placing it logically under autonomy (Varela, 1981: 14), this distinction can best be understood when one recalls the underlying motives. Autopoiesis was and is sought to be the central concept defining a living system and providing an answer to Schrödinger's question. The battlefield is biology, not any other scientific discipline. Regardless of this distinction, the importance of the notion of autonomy, 'that is, the assertion of the system's identity through its internal functioning and self-regulation' (Varela, 1978: 77), can be seen throughout Varela's work, from its centrality for the autopoiesis concept, where it was used to position forces against genetic determinism (Fleischaker, 1988: 39), to his work on identity and cognition. In order to understand the connection between the living and its identity construction through (self-)cognition, autonomy as the 'fundamental' condition of the living needs to be taken into account (Varela, 1997: 73). Glanville notes, in an obituary for Varela, that the assertion of autonomy was not only a scientific matter for him, but also a personal one, reflected in the turn towards Buddhism and Continental philosophy (Glanville, 2002: 67). Varela himself admitted that creating his 'own original science' (Varela, 1996: 411) was a matter of staying true to himself, his past and his roots. Again, no distinction is drawn without a motive.

Autonomy is tied to the third notion, that of *closure*. Axiomatically, Varela formulates his

closure thesis: *Every autonomous system is organizationally closed* (Varela, 1978: 79). Why should that be substantial? Because the process of identity construction of an autonomous system is always one of closure from its environment, and to close means to establish a perfectly contained circular process producing itself, thus constituting its autonomy (Varela, 1997: 73). The process of closing can be sketched by Kauffman's visualization of an arrow bent towards itself and, therefore, pointing to its beginning (Kauffman, 1987: 54). Closure thus defines a unity that is produced by the contained network of interactions producing themselves 'as a unity in the space in which the components exist by constituting and *specifying the unity's boundaries as a cleavage from the background*' (Varela, 1981: 15; emphasis added). According to Fleischaker (1988: 38), Varela replaced the original closure adjective 'organizational' with 'operational' somewhere around 1982, but apparently, he seems to use the terms interchangeably. This might be due to his rooting in biology and that therein 'organizational' and 'operational' is synonymous or arise at the same time. However, through closure, intrasystemic coherence without the need for central control emerges, whereas at the same time, closure from an environment paradoxically implies dependency on that environment (Varela, 1997: 73–78).

Closure now is leading to the last notion mentioned previously, that of *self-reference*. From what has already been said, it becomes clear that any autopoietic system, in order to maintain its autopoiesis (organization), needs some form of closure (distinction) from its environment, thus stating its autonomy from this environment. Closure, autonomy and, in fact, autopoiesis require a production of a boundary through the boundary, that is, presently drawing a distinction with the help of past drawings of a distinction. Only then the system can interact with its environment without loss of identity (Varela, 1981: 15–17). In other words, autonomous, self-producing systems construct their environment (draw a distinction) in order to be and stay themselves. Autonomy as self-rule through closure and self-production (autopoiesis) leads to the insight that 'the rules of operation are all

self-contained, there is no possibility of referring to the outside from inside the system' (Varela, 1997: 73). That is, the autonomous system can only refer to itself; the arrow has been bent inwards and created a circular loop of self-creation. Luisi (2003: 51) terms the autonomous identity of a closed autopoietic system as 'auto-referential', producing its own rules of existence. In such forms of closure, component interactions become fixed-point solutions¹ of the autonomous system, where the fixed points are *Eigenbehaviors* or self-determined behaviors expressing systemic invariances against the environment specified by the system itself (von Foerster, 2003). The system exhibiting such *Eigenbehaviors* becomes 'aware' of itself through cognizing the drawing of the distinction between itself and its environment and the understanding of this distinction as an indication what the system is and what it is not. As Kauffman (1987: 53) puts it, '[t]he self appears, and an indication of that self that can be seen as separate from the self. Any distinction involves the self-reference of "the one who distinguishes"'. Therefore, self-reference and the idea of distinction are inseparable (hence conceptually identical).¹ So the 'know-it-self' is crucial for identity construction, for only through this self-knowing the self thus constructed can detect the presence of something foreign, and in fact, itself is the only point of reference for a closed system (autopoietic system, autonomous system) to build discriminations of what it is not. Through self-reference, a system creates its own teleology and, by doing so, reproduce itself indefinitely (Weber and Varela, 2002: 120). To conclude on self-reference, it has multivalued meanings, appearing (i) as a procedure applied to itself, (ii) a reflexive domain of systemic invariances and (iii) a fixed point of a system's *Eigenbehaviors* (Soto-Andrade and Varela, 1984: 18).

All these notions—autopoiesis, autonomy, closure and self-reference—are intimately connected. In Spencer Brown's words, they all can

be confused. All of them are arising from the main motive of Varela's research, to give an answer to Schrödinger's question about the nature of life. In order to answer this question, one has to cope with issues of self-referencing, self-producing and self-closing systems that are autonomous with respect to their environment. When starting his journey towards the answer in the 1970s, there was no foundation to conceptualize these kinds of systems in a scientific manner. The turn from cybernetics towards computationalism, signified by John von Neumann, in Varela's view led the field further away from dealing properly and respectfully with the question (Varela, 1986: 117). The road Varela was taking instead was a journey into the heart of paradise: 'That paradise is something very concrete, founded on the logic of self-reference, on seeing that what we do is a reflection of what we are' (Varela, 1976: 31). As beautiful and aesthetic as his words may appear to the reader, the task sketched by them led Varela to formulate a mathematical foundation of autonomous systems. It is the main argument of this contribution that he not only succeeded in doing so but also provided a formal grounding for those sciences dealing with self-referential, self-producing and self-closing autonomous systems. We will retrace his steps and dive into the heart of paradise Varela found for him and us.

FROM THE LAWS OF FORM TO A CALCULUS FOR SELF-REFERENCE

Whenever there is an autopoietic, that is, self-producing entity, this entity needs to close on itself, draw a distinction between the environment and itself and, through that, indicate what it is and what it is not. This whole circular process is autonomous, without reference to anything outside itself, and it is self-referential. The hardships with self-reference have all the same root: the distinction between actor or operand, and that which is acted or operated upon, collapses. The system thus constituted is of its own making, its own product and producer. This autopoietic ontology gives rise to a severe epistemological crisis, the 'minefield' as stated

¹ Fixed points are solutions to an equation of the form $f(x)=x$. Varela (1979a: 171) gives an illustrative example by Heinz von Foerster of a fixed point in the linguistic domain: "This sentence has thirty-three letters", where the sentence S has its only fixed point solution for $n=33$ letters, that is, the form of the sentence is identical with what it indicates.

previously: the introduction of the observer as being inseparable from the observation, even worse, the observer observing and thus creating both the observer and what is observed. Varela framed the problem like this: the peculiarity of such systems 'lies in being self-indicative in a given domain, in *standing out of a background by their own means*, in being autonomous as the strict meaning of the word enounces' (Varela, 1975: 5; emphasis added). For Varela, the problem of formulating a conceptual foundation for this type of system could only be solved by basing it firmly on its self-indicational grounds. Precisely because of that, his attention turned to George Spencer Brown, the *Laws of Form* and the calculus of indication (Spencer Brown, 1969).

Much has been written about George Spencer Brown and the *Laws of Form*. Kauffman (2002: 50) sees it as 'a lucid exposition of the foundations of mathematics. It embodies a movement from creativity, to creation, to symbol, to system and language and thought and self.' The initial motivation of Spencer Brown, however, was of a very earthly matter: the scheduling of trains in tunnels (Marks-Tarlow *et al.*, 2002). The algebraic equations he developed sometimes needed to refer to themselves, resulting in paradoxical solutions in which the Aristotelian *tertium non datur* could not hold. The results oscillated between true and false or, in Spencer Brown's terms, marked and unmarked. In order to deal with this, Spencer Brown worked backwards in mathematics, beyond its Boolean foundation in logic, and arrived at 'seemingly the most basic symbol system possible, involving only the void and a distinction in the void' (Robertson, 1999: 44). The apodictic start into the first chapter of *Laws of Form* begins with the idea of distinction, and that by drawing a distinction, an indication is made to what is marked or named by the distinction. Both distinction and indication come together, bring forth each other (Spencer Brown, 1969: 1). When Spencer Brown speaks of the form of distinction, and that this is to be taken as the form, the notion of 'form' reflects the Aristotelian distinction between form and matter. A form is what bounds 'things'; that is, the form constitutes their boundary and in fact themselves—in Varela's case, the autonomous system, and in

Spencer Brown's case, the very fundament of mathematical reasoning (Russell, 1946: 162). The notion of boundary thereby is also very basic: In drawing a distinction, a space with two separated realms comes into existence, and in order to reach one side from the other, one has to cross the boundary. The definition of distinction as perfect continence strongly resonates with the ideas of closure and autonomy mentioned before. The motive of distinguishing, for example, a living cell from its environment is to uphold its autopoiesis, thus giving a value to it that differs from the value of its environment. From these very basic ideas of distinction, indication and continence, two situations almost naturally emerge, building the basic axioms of the calculus of indication. The first situation occurs when a distinction that has been drawn is indicated again. This is a mere description of what is already obvious and not changing anything, like writing the name of the distinction on its outside: the law of calling—'the value of a call made again is the value of the call.' The second situation is sometimes not so easy to comprehend. Spencer Brown develops a bridge between his two axioms out of the law of calling and what he stated before. Remember that a distinction is always drawn with a motive; that is, it is of a value that differs from what is not distinguished. Whenever this is the case, a distinction is drawn in order to name the differing values. If a call is made, the call is of value and can be recalled over and over again without changing its value. To call is to draw a distinction, thus creating a boundary and crossing it to the side that is of value, naming it as valuable or, in other words, as marked. The second situation now is that of crossing the boundary again, and that crossing can only lead into the unmarked state: the law of crossing—'the value of a crossing made again is not the value of a crossing.' These two axioms are representing the Janus-faced nature of distinctions. Distinctions can be both descriptive and merely naming what is distinguished, as well as operational: draw a distinction (Schiltz, 2007: 11).

The notation of the calculus of indication is also very basic. The mark of distinction, referred to as the cross, is a diagonally cropped rectangle with its right vertical side denoting

the distinction and the upper horizontal side denoting the indication. The law of calling can then be pictured as follows:

$$\neg \neg = \neg \quad (1)$$

The law of crossing translates into the following:

$$\neg \neg = \quad (2)$$

The unmarked state is not given a sign but denoted by a blank space. With this one symbol and the absence of it, and the two axioms, the entire calculus of indication develops.²

Spencer Brown stopped his work when self-reference was encountered. Chapters 11 and 12 of the *Laws of Form* deals with what he calls 're-entry', that is, what happens when the distinction is inserted in itself, the oscillations of values from marked to unmarked. This behavior arises from the following expression:

$$f = \bar{f} \quad (3)$$

When f is taken to be the marked state and inserted on the right-hand side, what follows from the law of calling is a cancellation of that side, thus resulting in the paradoxical situation where the marked state is identical or, as George Spencer Brown sees the equal sign, can be confused with the unmarked state of the void:

$$f = \neg \Rightarrow \bar{f} = \neg \neg = \Rightarrow f = \quad (4)$$

If the unmarked state is now inserted, the result is the marked state and so on. For every odd number of crosses or marks, there appears to be some-thing from no-thing (Robertson, 1999: 255); in fact, both are identical in the form of re-entry. George Spencer Brown's 'most outstanding contribution' (Varela, 1979a: 138) was

the realization that this behavior was in fact equivalent to that of imaginary numbers, that is, numbers that have an imaginary part i of the form $i^2 = -1$. Although in normal mathematics, these numbers were interpreted as being 'orthogonal' to the real numbers, Spencer Brown interpreted his re-entering expressions as oscillations *in time*. Whereas the calculus produces space by the injunction to draw a distinction, by re-entering the calculus into itself, it produces time. Although Spencer Brown finished his work there, Varela decided to start his journey right here.

A calculus for self-reference (Varela, 1975) is an extension of the calculus of indication, motivated by Varela's work on living systems and how to formalize their key characteristics, namely, their self-referential nature. In a way, this work is a prime example of how Spencer Brown's *Laws of Form* can be extended and, in fact, applied to other realms than mathematics. The emphasis on and embracing of self-reference, driven by Varela's motive of finding a conceptual foundation for the autonomous systems he was dealing with as a researcher, changed the indicational perspective insofar as self-reference, the 'end' of the *Laws of Form*, is now as constitutional for it as the primary distinction Spencer Brown made. Robertson (1999: 53) noted that this 'is ... an explicit creation of a new calculus in which self-reference will be the core.'

Varela (1975: 5) himself describes self-reference as 'awkward' and that 'one may find the axioms [of a calculus for self-reference] in the explanation, the brain writing its own theory, a cell computing its own computer, the observer in the observed, the snake eating its own tail in a ceaseless generative process.' This strengthens the claim made before that self-reference, self-production, circularity and autonomy are intertwined and emerge from each other. Whenever there is closure, that is, a distinction, that what is constituted by this closure indicates itself and thus refers to itself and produces itself, and it does so without reference to anything other than itself: it does so autonomously. This restated definition of autonomy is important because Varela uses this notion as the core of his new calculus: 'Let there be a third state, distinguishable in the form, distinct from the marked and unmarked

² The reader is advised to work through *Laws of Form* herself and spend "delightful days" (Beer, 1969: 1329) with a pen and some paper, watching the world unfolds itself in front of one's eyes.

states. Let this state arise autonomously, that is, by self-indication: Call this third state appearing in a distinction the *autonomous state*' (Varela, 1975: 7; emphasis added). Like autopoietic systems arise autonomously, that is, by themselves through the process of the network of interactions in their domains, so is the case with the calculus of these systems. It is a state in the form, arising from the indicational perspective when turned towards itself. The notation Varela has chosen for this third state in the form refers to Spencer Brown's rewriting of

$$f = \overline{fa|b} = \overline{\overline{a|b|a|b|a|b} \dots} = \overline{a|b} \quad (5)$$

Here, the re-entry of the distinction between *a* and *b* is inserted under *a*, denoted by *f* and resulting in an infinite nesting of crosses indicating the distinction between *a* and *b*.³ The *f* can be confused with a horizontal 'hook' attached to the bottom of the cross containing the distinction between *a* and *b*, pointing to where the whole expression should be inserted. This 'self-cross' resembles the ancient symbol of the Uroboros, a serpent eating its own tail:

$$f = \overline{f} = \overline{\quad} = \text{Uroboros} \quad (6)$$

In his *Arithmetics of Closure*, Varela together with Goguen develops another form of graphics representing both indicational as well as self-indicational expressions: the tree (Varela and Goguen, 1978: 308–310). The expression of

$$\overline{a|b} \quad (7)$$

³ This is done by applying consequences C1, C4, C5 and the initial J2 of the primary algebra (Spencer Brown, 1969: 55).

can thus be written as

$$\overline{a|b} \quad (8)$$

denoting the containment operation at *b* and the placement of *a* under its cross and the cross containing it and *b*. Similarly, the re-entrant expression (5) is then transformed into

$$\overline{a|b} \quad (9)$$

Consequently, the re-entrant expression (3) is then perfectly symbolizing the process of closure, the arrow bent towards itself:

$$\overline{a|b} \quad (10)$$

The autonomous state, as the third state in the form, represents a *description* of the re-entering expression, that is, of itself, as well as an *injunction* to re-enter autonomously from any environmental conditions and thus create itself by itself. Just as the mark of indication, the cross having two meanings—to name it as a cross (a distinction, a cross) and to demand a crossing (to draw a distinction, cross!)—the mark of re-entering indications, the Uroboros has two meanings: to name the spatial pattern as re-entry (an indication entering its own indicational space, a self-cross) and to demand re-entry (to re-enter one's own indicational space, self-cross!). There are snakes all the way down (Kauffman, 2002: 58), and they have to bite their own tail. The form of re-entry can thus be viewed as an *Eigenform*, a self-determined or autonomous form, a solution to a non-numeric equation describing the act of distinguishing oneself from

its environment. Varela (1979a: 125) himself makes the connection to time Spencer Brown already mentioned and exposes the double nature of self-reference and its confusion of operator and operand: 'We have to pay attention to the fact that the double nature of self-reference, its blending of operand and operator, cannot be conceived of outside of time as a process in which two states alternate. . . Both aspects are evident in the idea of autopoiesis: the invariance of a unity and the indefinite recursion underlying the invariance. Therefore we find a peculiar equivalence of self-reference and time, insofar as self-reference cannot be conceived outside time, and time comes in whenever self-reference is allowed.' This is exactly the conceptual foundation Varela needed for the question posed by Schrödinger. Life itself is autonomous; it arises out of itself and cannot be reduced to anything outside or inside its own creative, repetitive loop, where end products are fed back into the system as new points of departure (Marks-Tarlow *et al.*, 2002).

In the original article (Varela, 1975), after developing and proving its initials, consequences and theorems, Varela exemplifies several uses of his calculus. First, because every higher-order expression, that is, re-entrant expressions, can either be reduced to the autonomous state or the unmarked state (second-order expressions like (3)) or reduced to one simple expression, that is, marked, unmarked or autonomous (third-order expressions like (5)), the calculus 'not only shows that all self-referential situations can indeed be treated on an equal footing as belonging essentially to one class, but also shows a way to decide when an apparently self-referential situation is truly such' (Varela, 1975: 20). One application developed by Kauffman (1978) is the simplification of digital circuits, where the calculus for self-reference is used for valid network transformations without changing their global properties.

Second, the self-cross can be interpreted either as the spatial form of re-entry, thus providing an economic notation for an infinite arrangement of nested crosses, or as the temporal form of its unfolding dynamic, thus giving rise to an oscillation in time between markedness and

unmarkedness (Varela, 1975: 21). This has been further developed in *Principles of Biological Autonomy* (Varela, 1979a) and *Form Dynamics* (Kauffman and Varela, 1980) with the notion of 'waveform arithmetic' and is in fact a new extension of *Laws of Form*. The idea is that the re-entrant expression (3), as the basic form of re-entry and interpreted as a dynamic unfolding, gives rise to two different oscillations, depending on the initial value (either marked or unmarked). These so-called waveforms *i* and *j* can be written as

$$i = \overline{m} \overline{n} \overline{m} \overline{n} \overline{m} \overline{n} \overline{m} \overline{n} \dots \quad (11)$$

$$j = \underline{n} \underline{m} \underline{n} \underline{m} \underline{n} \underline{m} \underline{n} \underline{m} \dots \quad (12)$$

with *m* denoting markedness (the cross) and *n* denoting unmarkedness (the void). What is interesting about these waveforms is that they are fixed points for the marked state combined to

$$ij = \overline{\overline{m} \overline{n} \overline{m} \overline{n} \overline{m} \overline{n} \overline{m} \overline{n}} \dots \quad (13)$$

which according to the law of calling can be condensed to one cross. Thus, the temporality of the waveform, arising from the spatial form of the self-cross and the autonomous state it signifies, collapses into the spatial form of the cross and the marked state it signifies: 'The fixed points of the operator represent the spatial view of the oscillation, while its associated sequences represents the temporal context' (Varela, 1979a: 153). This sheds light on the issue of infinity and self-reference, because self-reference is the infinite in finite guise (Kauffman, 1987: 53) and *vice versa*: infinity, in autonomous systems, is a temporal unfolding of a finite Eigenform, that is, the system's invariances against its environment. Infinity in self-reference does not grow across all borders, which is always a problem in mathematics when dealing with real systems, but rather grows 'within': *f* embodies a copy of itself within itself, thus stating its autonomy against its environment over and over again.

Third, the calculus for self-reference can be interpreted as a three-valued logic, just as Spencer Brown's calculus of indication can be interpreted as propositional logic. Such a logic of self-reference abandons the *tertium non datur*

of two-valued logic systems, as a can be $\neg a$ in autonomous systems (Varela, 1975: 21). Varela cites Günther (1967) and his interpretation of a third value as time, however, claims that the calculus for self-reference is, just as the calculus of indication on which it rests, on a level deeper than logic. Space is created by drawing a distinction and thus making an indication. By inserting the distinction in the space it indicates, that is, by self-indication, time emerges, which can be confused with self-reference. There is no logic involved on this level; logic is something that is developed from this with a motive. Varela fully developed a three-valued logic in a separate work (Varela, 1979b), but his main intention was beyond logic. Both his newly developed logic and the calculus for self-reference carrying it were seen by him as linguistic carriers for describing, modelling and simulation of self-referential systems. Varela's own motive of drawing his distinctions the way he did is indicated once again by himself, and this motive is the application of the calculus in order to come to terms with self-referential, self-closing and self-producing autonomous systems and, ultimately, pay due respect to the riddle of the living.

These few remarks show that, just as the calculus of indication, Varela's calculus for self-reference (the calculus of self-indication) can be interpreted in many ways. It has to be noted here, however, that it is neither a three-valued logic nor a theory of autonomous systems nor a guide in self-referential situations. It can be all of that and more, if there is a motive to confuse it for that. What it actually is might best be described as a formal notation for 'the relation between a *form* (or figure) and its *dynamic unfoldment* (or vibration)' (Varela, 1979a: 113; emphasis added) and an abstract concept of a system whose structure is maintained through the self-production of and through that structure. In the words of Kauffman (2002: 57–58), it is 'the ancient mythological symbol of the worm ouroboros embedded in a mathematical, non-numerical calculus.'

CRITICISMS AND A REBUTTAL

The simple idea, that by drawing a distinction a universe is created and that by inserting this

distinction in the space indicated by it the autonomy of the living arises, received much praise as the quotations before have shown. Of course, criticism also is abundant, and the most severe arguments against the 'Flaws of Form' (Cull and Frank, 1979) and the 'awkward calculus of awkwardness' (Kaehr, 1980) need to be encountered here. These arguments can be summarized as non-originality, notational ambiguity and the problem of infinity.

The arguments against the originality or non-originality of the indicational perspective rest on the claim that nothing new is introduced to mathematics by it; in fact, the standard Boolean algebra is only developed from a different viewpoint without much merits. Also, the blending of operator and operand with the symbol of the cross is supposed to give rise to ambiguity in calculation (Cull and Frank, 1979). The first mistake this type of critique is making is the confusion of the indicational perspective with a form of logic. The calculus of indication, however, is not a logical calculus. Kauffman and Solzman (1981: 254) sees it as 'is an empirical case study of the consequences of distinguishing, begun as simply as we (and presumably Spencer-Brown) know how.' The motive of Spencer Brown to draw his distinctions the way he did was the search for the simplest beginning, an inquiry into the nature of arithmetics, into the 'sociology of numbers' before any number is constructed. The geometry of the arithmetic developed in *Laws of Form* has no numerical measure; it is as basic as one can get. It provides a view on how 'things' of whatever kinds are constructed, come into being and merge into another. It is the foundation of anything, of some-thing from no-thing, of becoming being (Schiltz, 2007: 11). Boolean algebra cannot deliver that, because it is 'already' constructed from something deeper than itself. In other words, Boole's system is not autonomous. The notation Spencer Brown introduced, which in turn is 'just' a continuation of the idea of a distinction drawn in a plain and thus establishing some indicational space of perfect continence, is giving rise to a very clear system of markings and transformations. It is as unambiguous as math can get, and the 'ambiguity' from the blending of operator and operand is at

best a virtual one, because there 'are neither operators nor operands in the primary arithmetic; just marks, expressions and changes due to the transformation rules of calling and crossing. We can view the mark of distinction as an operator *if we like to*' (Kauffman and Solzman, 1981: 254–255; emphasis added), that is, if we have a motive to do so and this is saying more about the one drawing a particular distinction than about the notation of distinction. However, as Kauffman notes against Cull and Frank (1979: 202), mathematics is not independent from notation, and notation is nothing near 'insubstantial'. The invention of the zero or negative numbers or, especially fitting when dealing with the calculus of indication, the imaginary numbers are prime examples of how notational novelty enables mathematics to tackle problems formerly thought unsolvable.

The arguments against the careless use of infinities arising from re-entry, especially in Varela's calculus for self-reference, have been stated most severely by Kaehr (1980), a former student of Gotthard Günther. He claims that the dream of second-order cybernetics, and building on that also of Varela, would be to turn a straight line into a circle in infinity. Through the introduction of self-reference, a series of markedness turned unmarkedness and so on grows, as a recursive expression, to indefinite length. According to Kaehr and his motive, which is with the realization of artificial intelligence, this is thinkable but not operational, that is, self-reference cannot be constructed in the real world like that. As has been argued, infinity in the calculus for self-reference, through the re-entry of a distinction in its own indicational space, is not growing 'beyond borders' but reaffirms the autonomy of what is self-indicated against its environment. Expression (5), for example, reaffirms that there is an *a* in the context of *b* in ever deeper spaces, that is, *time and time again* thus turning it into a stable, self-determined value. It might be a motivational error to suppose that this could not be realized, which might be the case for artificial intelligence, but for any other intelligence, for any other living system, it is not problematic at all. It happens quite effortlessly all the time around us. This motivational error

confuses a problem in mathematics—how to construct an infinite series that bites itself in its tail—with a problem beyond mathematics (Kauffman, 1996: 297–299).

The rebuttal of the criticisms launched at the indicational and self-indicational perspectives might appear as a form of dogmatic romanticism because both Spencer Brown as well as Varela use a very lively and colourful language, attractive to some, mystical blabbering to others. In fact, a subtle criticism throughout the reminiscence of the *Laws of Form* and the idea of self-reference that captured Varela's mind is aiming at the way these ideas are presented (Kauffman and Solzman, 1981: 254, Kauffman, 1996: 299). Both have turned, in their writing and thinking, to Eastern philosophy. The *Laws* are inherently Daoist, and Varela, very early in his life, became a Buddhist. Whatever impacts these spiritual leanings might have had on the ideas, and they surely have as nothing is distinguished without motive, the reasons both Spencer Brown and Varela had to resort to the language they were using is given in the problem they were faced with. When going back to the simplest thinkable origin, in Varela's case, 'to find the simplest possible way to symbolize a reality which explicitly includes self-reference' (Robertson, 1999: 53), one encounters the limits of thinking which are drawn in language. Glanville (2002: 70–71) says it very precisely: '[W]hen I accept that I have reached a fundamental—that there are no more distinctions—I have drawn the re-entrant distinction "that there are no more distinctions to be drawn". And, when I accept that I have reached the Universal, I accept that I have drawn all distinctions, by drawing the re-entrant distinction "that there are no more distinctions to be drawn"'. This is the point where probably both Spencer Brown and Varela realized that the 'form' is a symbol for the world that cannot be referred to from anywhere else but itself. Everything that follows rests on some-thing from no-thing, that is, from referencing itself from within itself. In a Foucauldian sense, when you take away all the masks of reality, you end up with what reality is: a pile of masks referring to themselves. The *Laws of Form* and their calculus for self-reference are anti-essentialist to the bone

or, better, *to the form*. The form marks the end of language and description as well as the start of it. It is an offer for the 'empirically minded scholar' to follow what can be taken out of the form 'with an open heart' (Kauffman and Solzman, 1981: 256) and apply it to the seeming irrationality of the real world. This offer for practice and application, that immediately follows from the basic ideas of indication and self-indication, which are in themselves empirically valid and experienced all around in the living, is probably the most overlooked part in Spencer Brown's and Varela's work.

Quite contrary to the arguments against and for the language used in Varela's elaboration of the calculus for self-reference and his theory of autonomous systems, the major breakthrough in biology, the science dealing with Schrödinger's question, did not occur because of its overtly mathematical nature. The research programme Varela laid out in his *Principles of Biological Autonomy* was largely unanswered by mainstream science, probably because of the challenges it posed to so many established concepts, 'perhaps too many to be widely embraced, perhaps too far ahead of its time' (Krippendorff, 2002: 95). What is also striking to note is that in later years, Varela did not explicitly work on or use his formal system for self-reference. It is rather more implicitly resonating in the language he uses than in any formal description. Maybe it was too far ahead of the time, even for him. Maybe it was a Wittgensteinian ladder he had to take in order to climb to the playing field of self-reference, and that could be thrown away afterwards. One can also argue that his Buddhist worldview that he found for him in the 1970s is the bridge over which he came to the extended calculus and further to enaction theory (Brier, 2002: 81). Thus, the calculus for self-reference and the *Laws of Form* can be on one side of the distinction, whereas his later work is on the other. As we know, both can be confused and might well prove to be identical in the form. Whatever may be the case, the core motive of Varela (1979b: 141) should not be forgotten: 'I have taken the Calculus of Indications as a starting point in an attempt to produce adequate tools to deal with self-referential situations. Self-reference is, of course, of great

historical importance; it was responsible for a major crisis in mathematical thinking at the turn of the century. More recently, with the development of cybernetics and systems theory, other aspects of self-referential situations have become apparent, namely, the fact that many highly relevant systems have a self-referential organization.' The key problem when dealing with 'system-wholes' that produce themselves autonomously through the production of a boundary is that of self-reference. In developing a rich body of formal foundations for self-reference, Varela in fact provided the grounds on which any theory of autopoietic systems can and has to stand. Autopoietic systems, as Schiltz points out, have their own self-production as the Eigenform of their 'basal unrest, the abbreviated expression of the system's concern with getting around its non-identity' (Schiltz, 2007: 21), that is, reaffirming its autonomy in relation to an environment while constantly crossing the boundary of what the system is and what it is not. Varela's work on self-reference, his many-faceted self-indicational perspective, does provide a grounding not only 'for every description of any universe' (Varela, 1975: 22) but also on all self-referential systems of whatever kind.

THE PRAXIS OF SELF-REFERENCE

Ten years after Varela's untimely death and more than 35 years after *Calculus*, the great promise of the logic of paradise is still to unfold. There are some hints where we can find traces of it. These hints stem from psychology, Varela's own theory of embodiment, and the playful use of self-referential logic in social systems.

In psychology, it was Gregory Bateson who introduced the notion of 'double bind' while studying the communication patterns in families of schizophrenics (Bateson, 1972: 271–278). A double bind exists when in a continuing pattern of communication, that is, a situation with no exit option, a certain behavior B is produced and, in turn, the opposite behavior $\neg B$ is also produced.

This situation is clearly self-referential and can be depicted as

$$\boxed{B} \quad (14)$$

In the classic situation of the mother demanding, in a verbal form, 'love me', while at the same time, in body gestures, signalling 'do not love me', the resulting Eigenbehavior is of the kind 'love me love me not love me not love me not...' Expression (14) is clearly neither 'love me' nor 'love me not'; it is, in this specific situation, an undesired autonomous state of the double bind that produces a pathological context (Varela, 1979a: 198). For example, for family therapy, it is clear then that such a situation cannot be solved by focusing on who loves who or who does not but on the new state that is emerging as an Eigenbehavior of the situation. As a waveform, it resembles *i*, and it should be remembered that in combination with *j*, it constitutes the fixed point for the marked state. If the initial marked state is 'love me', there is hope that by combining the two waveform patterns, that is, by contrasting 'love me' with 'love me not' *simultaneously*, the marked state, that is, 'love me', should occur. In a therapeutic context, this would imply confronting the mother who says 'love me' with 'would you want me not to love you'. This is clearly paradoxical as the entire unfoldment of the situation; however, it would create the fixed point necessary for 'love me' to stabilize itself. In a very similar manner, Ronald Laing, having some connections to Spencer Brown, described the structures of human relations as inextricable knots (Laing, 1970). The famous 'Jack and Jill' story describes a situation where, for example, Jack is hurt by thinking that Jill thinks he is hurting her by being hurt by thinking ... and Jill is hurt by thinking that Jack thinks she is hurting him by being hurt by thinking ... The oscillation of both feeling hurt by the assumption why the other is feeling hurt is the Eigenbehavior of Jack and Jill, which is neither Jack and his feelings nor Jill and her feelings but the

insertion of Jack is hurt by the fact that Jill is hurt into Jack's hurt:

$$\text{Jack and Jill} = \boxed{\text{Jack is hurt} \mid \text{Jill is hurt}} \quad (15)$$

To assume the 'waveform solution' for this situation would mean to question Jack why he loves Jill (hurt is marked, love is unmarked), thus giving rise to the marked state. It would be an interesting move to find a method for formulating social situations of self-referential nature as these kinds of stories, or 'riddles', as Spencer Brown did with Lewis Carroll's last sorites and apply the calculus on it (Spencer Brown, 1969: 123–124).

As stated before, Varela himself turned towards self-reference as praxis without naming either his calculus or the formal aspects of his work on autonomous systems. However, he stayed within his central notions especially that of autonomy when dealing with the nature of the living, and the reciprocal, that is, self-referring construction of identity, which becomes a (fixed?) point of reference for a domain of interactions. Whatever the parts of such an emerging system-whole may be, they can and must reference their interactions towards the system's identity (Varela, 1997: 73). Via this form of self-reference, the system, and Varela's (1997: 83) thinking, moves towards cognition that 'happens at the level of a behavioral entity and not, as in the basic cellular self, as a spatially bounded entity. The key in this cognitive process is the nervous system through its neuro-logic. In other words, the cognitive self is the manner in which the organism, through its own self-produced activity, becomes a distinct entity in space, but always coupled to its corresponding environment from which it remains nevertheless distinct.' From here, it is a small step to the notion of 'embodiment', the realization that life and cognition are embodied; that is, the system is the reflection of the distinction between system and environment within the system and thus, and only thus, the system 'knows' that it is not alone and ... *is*. In a scientific manner, Varela describes this as the

enmeshed network of nervous system, the body and their environment, and that the only proper understanding of life, cognition and mind is that of mutually embedded systems (Thompson and Varela, 2001: 423). In a more personal manner, taking notes of his experience with his liver transplantation, he reaffirms the organism's embodiment as a double quality of being a functional biological machine, but also by being lived ... by someone who draws a distinction, placing the body within the space indicated and at the same time switching between intimacy of your own self and a decentred intersubjectivity depending on whom you interact with that is not the self that ... (Varela, 2001). This can be notated as

$$\text{self} = \boxed{\boxed{\text{body, mind}} \text{ social, physical world}} \quad (16)$$

and which might be reduced to a simple expression as in

$$\text{self} = \boxed{\text{self}} \quad (17)$$

While reading his later work on enaction and embodiment, the formal background summed up in *Principles of Biological Autonomy* reverberates through the reader's mind. To formulate the problem of consciousness as embodied cognition and apply the formal notations developed by Varela does indeed indicate an entire new research agenda for the cognitive sciences and, in fact, pose a new path to answering Schrödinger's question.

However, there is more from self-reference than the field the calculus motivationally originated. The last field of praxis for the calculus for self-reference, therefore, is that of social systems. Luhmann (1995a) borrowed heavily from Spencer Brown and Varela, especially as regards the ideas of re-entry, autonomy, autopoiesis and closure in stating his theory of social systems. His use of the concept of autopoiesis as well as the indicational perspective systems as distinctions entering their own indicational domain resonate strongly with all that has been argued before. However, the application of the idea of autonomy through self-production and self-indication, that

is, through closure and self-reference, was not easily conceived. Varela himself was very sceptical when using, for example, the notion of autopoiesis for social systems. Instead of autopoiesis, autonomy for Varela (1981: 15) appeared to be the more exact property for a wider class of self-referential systems, including social systems: 'From what I have said I believe that these proposals are category mistakes: they confuse autopoiesis with autonomy. Instead, I suggest to take the lessons offered by the autonomy of living systems and convert them into an operational characterization of autonomy in general, living and otherwise. Autonomous systems, then, are mechanistic (dynamic) systems defined by their organization. What is common to all autonomous systems is that they are organizationally closed.' As has been already stated, both notions are so intricately connected that this may come across as hairsplitting; however, it is hairsplitting with a motive. As Luisi points out, the start of autopoiesis in biology was not easy; in fact, it was more regarded as being 'philosophy' rather than science. Moreover, the sudden upspring in the 1980s of autopoietic notions in social sciences 'not always in a very rigorous way' provided the entire set of ideas with a certain 'new-agey' shine, which probably was even deepened by the language of self-reference Varela took from Spencer Brown and Heinz von Foerster (Luisi, 2003: 50). Also, there were some heavy biases against the way Luhmann applied the notion of autopoiesis for social systems, foremost expressed by Mingers (1989, 2002). When reading through Mingers' arguments, however, his main objections are not likely to hold against what Luhmann actually did. Mingers objection can be condensed to two questions that he might regard as not properly solvable in the formal definition of autopoiesis: what do social systems produce? What are the components of social systems? We do not want to dive too much into the Mingers' arguments, as King and Thornhill (2003) have effectively rejected them quite thoroughly, but let Luhmann answer these questions himself: 'Social systems use communication as their particular mode of autopoietic reproduction. Their elements are communications which are recursively produced and reproduced by a network of communications

and which cannot exist outside of such a network. Communications are not “living” units, they are not “conscious” units, they are not “actions”. Their unity [of the autopoietic system] requires a synthesis of three selections: namely, information, utterance and understanding (including misunderstanding). This synthesis [unity] is produced by the network of communication, not by some kind of inherent power of consciousness, or by the inherent quality of the information.’ (Luhmann, 1986: 174) The network of communication is producing the boundary of the social system. It does so by processing a synthesis of

and climbed up the ladder of abstraction towards a general theory of autopoietic systems as the foundation of theories of psychic and social systems (Luhmann, 1986: 172). However, we are not so much interested here in Luhmann’s greatest efforts—namely, to free sociology from its humanist prejudices, its reliance on geography to indicate the space of society and the subject/object distinction that haunted many sciences (Luhmann, 1992)—but focus on his embracing of self-reference as key feature of social systems and the indicational perspective as best suited for developing a conceptual

$$\text{communication} = \boxed{\begin{array}{|c|c|c|} \hline \text{understanding} & \text{utterance} & \text{information} \\ \hline \end{array}} \quad (18)$$

whereas this form is the unity of the social system. This form, at the same time, provides for continuation of communication in the network of communication, thus continuing unity production. The form needs to show re-entry; that is, information enters utterance and understanding, thus re-entering the distinctions made. The unity of communication, the boundary of the social system, is produced through understanding the utterance of information as communication. Luhmann’s application of autopoietic thought was in fact not metaphorical at all but followed Stafford Beer’s ‘yo-yo model’ (Beer, 1970: 118)

foundation of them. Baecker (2006: 125) is drawing heavily on self-referential logic and the indicational notation for developing an array of forms for organization and management sciences (Baecker, 2006). For him, the form of distinction can be confused for ‘a simple device that consists in picturing operations within contexts, these contexts being operations that themselves call on further contexts.’ In drawing the form of the firm, Baecker assigns value not only to the different sides indicated but also to the re-entering operation itself, resulting in the following form

$$\text{firm} = \boxed{\begin{array}{|c|c|c|c|c|c|} \hline \text{product} & \text{technology} & \text{organization} & \text{economy} & \text{society} & \text{individual} \\ \hline \text{work} & & & & & \\ \hline \text{business} & & & & & \\ \hline \text{corporate culture} & & & & & \\ \hline \text{communication} & & & & & \\ \hline \text{ethics} & & & & & \\ \hline \end{array}} \quad (19)$$

where work, business, corporate culture, communication and ethics are distinct re-entering operations, each of differing value to what is re-entered with them. Although being more or less a useful heuristic to focus observation of the firm on different aspects of interests, Baecker (2006: 137) concludes that the next step for such a 'distinction view' of the firm would be 'to envision a kind of contextual mathematics as a means of analysing phenomena and constructing observations as a contribution to organizational practice.' This is clearly calling for a formal grounding, and Varela's work could play a significant role in this.⁴

All these forms of self-referential praxis show the richness of what springs from Varela's thinking and his move from first-order to formalized second-order descriptions. Luhmann (1995b: 52) himself demands such a move, going beyond what is to be found in the *Laws of Form* and the calculus of indication. We argue, and hope to have proven it as fact, that Francisco Varela achieved that move and made the first steps into a new world of formal descriptions.

OUTLOOK

From where we stand now, on the pinnacle of self-reference, looking down the valleys of paradise, what remains of Francisco Varela, what can and needs to be done? Varela's (1976: 31) dance with the world led him to the insight that in order 'to understand the whole of us and the world, we have to participate with the whole of us.' This call for participation resonates with the nature of the indicational perspective he deepened and widened to the calculus for self-reference and what he termed the *logic of paradise*, 'the possibility of a common survival with dignity of humankind' (Varela, 1976: 31). The praxis of self-reference, and his formal notation and system, can be valid across many scales of self-referential systems. The further formalization of these systems, and in fact, the scientific fields that are dealing with them,

⁴ The author of this paper has been involved in one of Dirk Baecker's conferences at the Zeppelin University in Friedrichshafen (Germany) on the "Mathematics of Form", involving Louis H. Kauffman and Matthias Varga von Kibéd.

is not hindered by any inaccessible barrier, not anymore. In Varela's (1981: 19) own words, 'there is no reason why there could be no mathematical theory of circular systemic processes. It surely entails some conceptual and formal readjustments, but no more so, say, than a rigorous theory of vagueness.' Two roads are laid out to be explored. The first is the formalization of theories currently 'under-formalized'. From a self-indicational perspective theories, for example, from the social sciences and psychology, the so-called 'soft' sciences could be reformulated in a notation close to the calculus for self-reference. For sociology, Luhmann and Baecker came nearest to that, and there appears to be fertile ground to continue the work. The second road is the formalization of empirical phenomena, that is, to apply the self-indicational perspective directly to social situations as in the psychological examples given. Can we, for example, use the calculus for self-reference in the way Varela envisioned it, to determine whether or not we see self-reference, and if so, if that is a contender for a system's Eigenform? If a system's Eigenform can be determined, we can understand what drives its identity construction and how this is done. Both roads will meet and cross each other several times, like an oscillation unfolding from the form of self-reference, the Uroboros itself. Theory that can be contrasted with practice can be contrasted with theory and so forth until we reach a better understanding of the self-referential systems we encounter. There is plenty of territory to be discovered in this land of paradise. To give Varela (1981: 22) the last words: 'So far, we have given substance to only a few item of this research programme. The rest is yet to unfold.'

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